Modeling Infant Free Play Using Hidden Markov Models

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***Abstract*— Infants' free-play behavior is highly variable. However, in developmental science, traditional analysis tools for modeling and understanding variable behavior are limited. Here, we used Hidden Markov Models (HMMs) to capture behavioral states that govern infants' toy selection during 20 minutes of free play in a new environment. We demonstrate that applying HMMs to infant data can identify hidden behavioral states and thereby reveal the underlying structure of infant toy selection and how toy selection changes in real time during spontaneous free play. More broadly, we propose that hidden-state models provide a fruitful avenue for understanding individual differences in spontaneous infant behavior.**

Keywords—Behavior Modeling; Developmental Science

I. Introduction

Infants are voracious explorers. They learn about the world by gathering information about people, places, and things. As infants explore, they acquire new skills that, in turn, drive the developmental cycle by expanding access to the environment and creating new learning opportunities [1]. Thus, a fundamental goal of developmental science is to understand infant exploration and factors that influence it.

Characterizing patterns of exploration, however, is tricky because every infant explores the environment differently. Moreover, individual infants can rapidly shift from one behavioral state to another, sometimes focusing on a single object or activity and sometimes interacting with everything in sight. Traditionally, developmental scientists focus on the amount of intra- or inter-individual variability in a behavior or task and not on its structure because researchers lack methods to identify structure based on variability [2]. Here, we provide a framework for identifying higher-order structure in observational data using Hidden Markov Models (HMMs).

Our goal was to test whether HMMs can reveal meaningful hidden structures in infant behavior. Specifically, we modeled spontaneous toy selection in 15-month-old infants. The dataset comprised 20-minute free-play sessions in a novel lab environment with toys and a stationary caregiver. The annotated data contain information about infant and toy locations and interactions. Figure 1 shows moment-to-moment toy interactions for one infant. Based on the annotated data, we asked whether HMMs usefully explain infant toy selection by identifying common behavioral states and whether commonalities/differences exist among inferred state sequences.

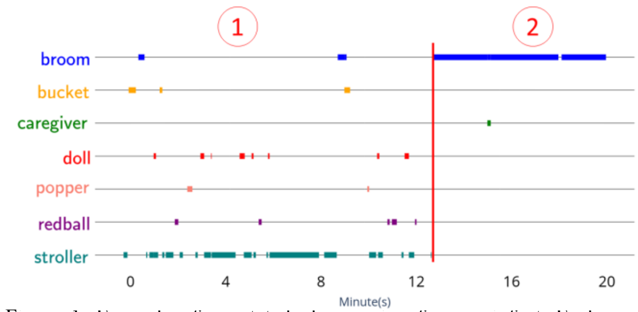


Figure 1. Example of annotated play session for one infant. Each row corresponds to a different toy and the x-axis denotes the 20-minute timeline of the session. Solid lines indicate interaction intervals for each toy. Time segment 1 shows an interval where the infant switched among a variety of toys; time segment 2 shows nearly complete focus on one toy.

HMMs provide many choices for modeling observable infant behaviors. We describe the design space we considered and the features, sequence segmentation, and data augmentation we found most effective. Our results demonstrate that HMMs provide a meaningful characterization of infants' behavioral states that inspire future investigation. We also describe general exploration patterns by clustering infants into groups based on their behavioral states and by identifying trends in the moment-to-moment progression. Overall, we demonstrate the benefits of modeling hidden structures in infant behavior.

We suggest that our HMM methods are useful for modeling hidden structures that help explain variability in other types of behavior (e.g., manual actions during object play). In addition, our methods can be extended to other modalities and contexts such as eye-tracking and group play. Our findings offer great promise for building increasingly sophisticated hidden-state models and their use in developmental science.

II. Related Work

Developmental scientists typically quantify infants' object interactions in terms of average durations, frequencies, or rates of switching among toys [3]. Despite immense individual differences in the timing of object interactions, researchers typically neglect such variability. A small number of studies used HMMs to reveal insights about gaming behaviors [4], test-taking behavior [5], and mother-infant interactions [6]. The only study to apply HMMs to infant free play behavior focused on recognizing pre-defined activities (e.g., crawling, climbing) from accelerometry data [7]. No studies used HMMs to identify higher-order behavioral states in infants' spontaneous behavior captured on video.

III. Experimental Method

To study infant toy selection and other types of free-play behavior, we collected and annotated data from play sessions in a laboratory playroom. Videos and annotations for each session are shared in the Databrary digital library (https://nyu.databrary.org/volume/140).

Participants. Twenty 15-month-old (± 1 week) walking infants (12 girls, 8 boys) were video recorded during 20 minutes of free play. Families were recruited from maternity wards in the New York City area and received small gifts as souvenirs of participation. All study procedures were approved by the NYU institutional review board.

Procedure and Playroom. Infants played in a large laboratory playroom ($4m \times 8m$) that was empty except for six toys (ball, "popper," baby-doll, child's broom, large red ball, bucket filled with small balls, and toy stroller) and a chair for the caregivers. At the start of each session, toys were scattered evenly around the room and placed in the same initial locations. Caregivers sat in the same location at the edge of the room and were instructed to let infants play freely. We filmed infants from four camera views: a fixed overhead view of the entire playroom; two fixed side views of the length and width of the playroom; and a handheld experimenter-operated camera for close-up views of hands and feet. The experimenter did not interact with infants or caregivers. Some sessions were slightly shorter than 20 minutes due to infant fussiness (*M* = 19.2 min, *SD* = 2.2).

Data Annotation. Two coders annotated infants' moment-to-moment interactions with the toys and their caregiver. Each coder scored half of the infants and checked the other half. All disagreements were resolved by discussion. Figure 1 shows a visualization of the annotation data for one infant. Coders scored toy interactions as the moment infants first touched the toy until they stopped touching the toy for at least 1s. Coders scored interactions with the caregiver during intervals when infants were in close proximity and actively engaging the caregiver. Infants frequently interacted with multiple toys and occasionally did not interact with either toys or their caregiver.

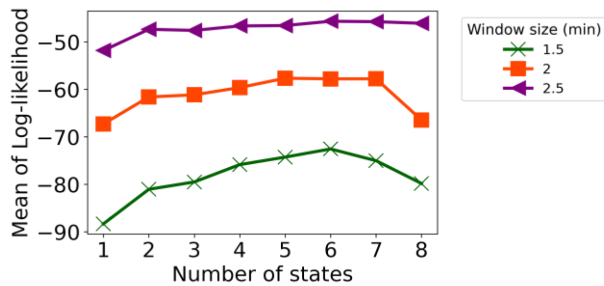


Figure 2. Result of Leave-One-Out Cross-Validation for different time discretizations (1.5-, 2-, 2.5-minutes). Note that log-likelihoods are not directly comparable for the three time discretizations. We see that for the 2-minute discretization, 5-states is the best value. See text for discussion of the other curves.

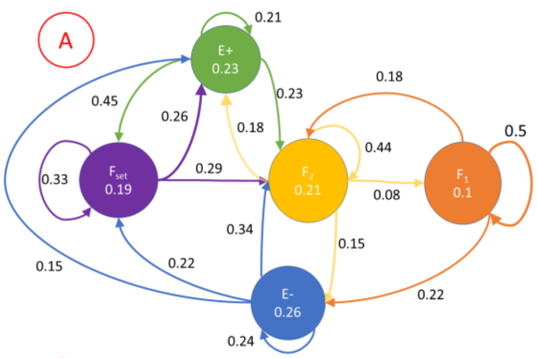
In addition to toy interaction annotations, the data were tagged with locomotion information. We used multiple camera views to track infants and toys, and recorded locations in laboratory-relative coordinates with *cm* precision. These annotations allowed us to estimate the distance travelled by infants during every play interval.

IV. Behavior Model Design and Estimation

Our primary goal in the model was to infer the behavioral states governing infants' toy selections. Because the internal states are not directly observable, we treated them as latent (i.e., "hidden") variables whose values evolve during the play session. In this section, we review HMMs and describe the design and model-selection choices used for our analysis.

Hidden Markov Models. HMMs [8] are a well-established class of probabilistic models of discrete-time dynamic systems (here infants) with hidden states (behavioral states) based on observation sequences of the data (toy selection data). In particular, an HMM is defined by a tuple (S, O, T, B, p_0) where S is a finite set of hidden states and O is the space of observations. The transition function T gives the probability, denoted $T(s' \mid s)$, of the system transitioning to state s' directly after being in state s. The observation distribution B gives the probability, denoted $B(o \mid s)$, of the system generating observation o when in state s. Finally, p_0 is the initial state distribution over S.

An HMM represents the generation of hidden states and observation sequences in a straightforward way. First, a state sequence $S_{0:T} = (s_0, s_1, \dots, s_T)$ is generated by sampling an initial state $s_0 \sim p_0$ and then generating the remaining sequence, in order, by sampling states from the transition distribution conditioned on the previous state, i.e., $s_t \sim T(\cdot \mid s_{t-1})$. Given the sampled states, the corresponding observation sequence $O_{0:T} = (o_0, o_1, \dots, o_T)$ is generated by independently sampling an observation at each time conditioned on the state, i.e., $o_t \sim B(\cdot \mid s_t)$. According to this generative process, the probability of a pair $S_{0:T}$ and $O_{0:T}$ is $\Pr(S_{0:t}, O_{0:t}) = p_0(s_0) \cdot \prod_{t=1:T} T(s_t \mid s_{t-1}) \cdot \prod_{t=0:T} B(o_t \mid s_t)$.



B

		To this state				
		F1	F2	Fset	E-	E+
From this state	F1	0.5	0.18	0	0.22	0.1
	F2	0.08	0.44	0.15	0.15	0.18
	Fset	0.06	0.29	0.33	0.05	0.26
	E-	0.05	0.34	0.22	0.24	0.15
	E+	0.06	0.23	0.45	0.05	0.21
Initial prob.		0.1	0.21	0.19	0.26	0.23

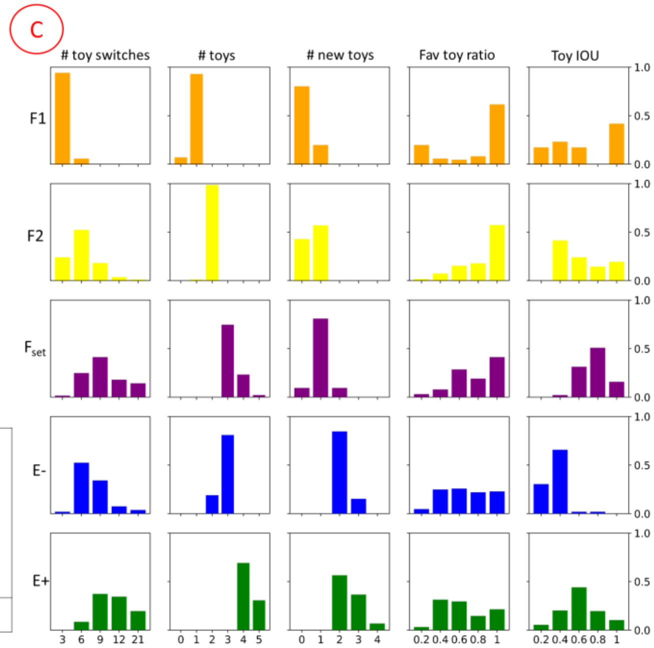


Figure 3. Estimated 5-state HMM for 2-minute windows. (A) Transition diagram: Each state is a circle with arrows pointing to successor states labeled by the transition probability. Only transitions with significant probabilities are shown. (B) Full state transition matrix with the initial state distribution in bottom row. (C) Observation distributions. Each row corresponds to a state and each column a feature. Each histogram shows the value distribution of the feature for the corresponding state. Features: *Number of toy switches*: number of times the toy set changes during the interval, *number of toys* interacted with in the interval, *number of new toys* (toys not played for the last 2 minutes) played during this interval, *favorite toy ratio*: duration of time that the infant plays with the favorite toy divided by the window size, *toy intersection over union* (IOU): intersection over union of two consecutive toy sets.

For N hidden states, the parameters of an HMM are the N^2 probabilities of the transition matrix T, the N probabilities of p_0, and the parameters of B, which depend on the form of the observations and observation model. Given a set of observation sequences, the maximum-likelihood parameters can be estimated using the classic Baum-Welch algorithm [9]. Given an HMM and observation sequence $O_{0:T}$, the Viterbi algorithm [10] can compute the most likely state sequence $S^*_{0:T}$ for the observations and the Forward-Backward algorithm [11] can compute the distribution over the states at each time.

Design Choices. To apply HMMs to our data we converted each play session into a temporal sequence of observations that captures the rich infant behavior. The original annotation time-scale of milliseconds was too fine-grained to capture meaningful behavioral states because the states persist over longer intervals. Thus, we discretized the play sessions into non-overlapping intervals that better match the time scale of behavior state changes. We selected 2 minutes as the window size based on visual analysis of play sessions and early exploratory data analysis. In the next section, we examine the effect of this choice on our findings. Using 2-minute windows, each play session is represented by ten discrete time intervals, where s_t and o_t denote the state and observation, respectively, for interval $t \in \{1, \ldots, 10\}$.

Each observation o_t is based on 5 meaningful features computed over the corresponding time window. Features were designed to capture key toy-selection characteristics:

# of toy switches: At any time point, infants could interact with 0-6 toys. This feature returns a count of the number of times the toy set changed during the interval, giving an indication of toy-selection focus.

# of toys: Number of toys interacted with in the interval.

# of new toys: A *new toy* is one not interacted with for at least 2-minutes prior. This feature computes the number of new toys per interval, capturing toy-selection novelty.

Favorite toy ratio: The *favorite toy* for an interval is the toy interacted with for the longest duration in that interval. This feature computes the interaction duration of the favorite toy divided by 2-minutes, which provides another measure of infants' focus on a single toy.

Toy intersection over union (IOU): This feature considers the set of toys interacted with during the current

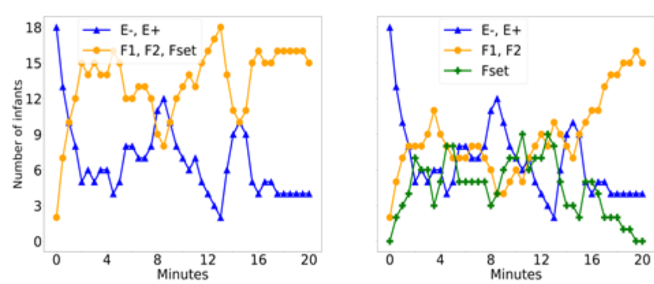


Figure 4. Left panel: Number of infants in focus vs explore states over the play session. Right panel: Number of infants in F1 and F2 vs Fset vs explore states over the play session.

interval I_t and the set of toys interacted with in the previous interval I_{t-1} and returns $|I_t \cap I_{t-1}|/|I_t \cup I_{t-1}|$, which is 0 when there are no common toys and 1 when the sets are identical. This feature also measures novelty in toy selection for the current interval.

We discretized each feature into 5 bins, so that each feature corresponded to a discrete variable with 5 possible values. This discretization was done by hand to ensure a non-trivial number of observations in each bin. After binning, each observation o_t is a 5-dimensional vector, where $o_t^{(i)}$ gives the discrete value of feature i for interval t. Given these observation features, the HMM observation model B is a Naïve Bayes [12] model over the features: $B(o_t \mid s_t) = \prod_i P_i(o_t^{(i)} \mid s_t)$, where each P_i is a multinomial distribution over the values of $o_t^{(i)}$ conditioned on s_t.

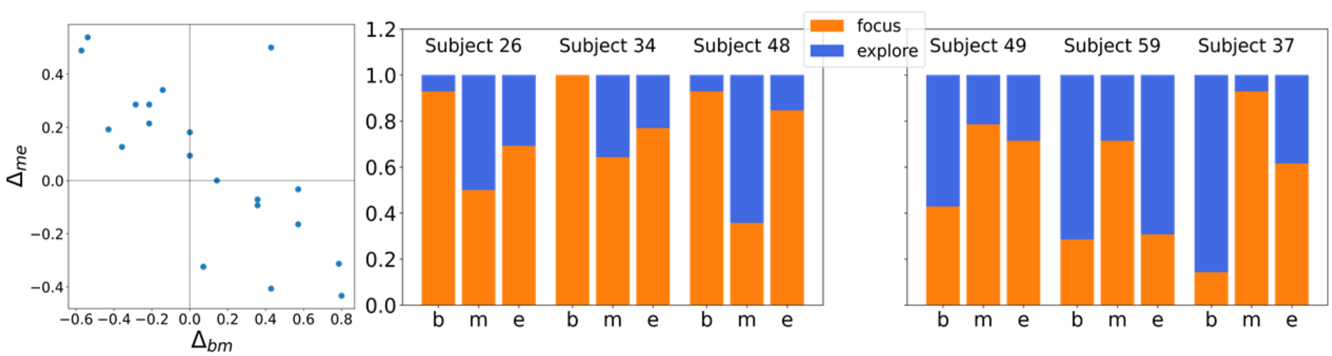
Parameter Estimation and Model Selection. Given the above design choices, we represent each play session as 10 sequences of observation-feature vectors. One potential concern is that infants' behavioral states are not aligned with 2-minute windows. Ideally, we would like to build models that are not overly sensitive to such misalignments. Thus, we augmented the dataset with 3 phase-shifts of the original data. We shifted the play session by 30, 60, and 90 seconds and computed three sequences based on the shifted versions. This yielded four observation sequences per infant for a total of 80 sequences.

Given a chosen number of N hidden states, we used the Baum-Welch algorithm to estimate the remaining HMM parameters. However, N is unknown in our application, which requires model selection. Because we have limited data and the sequences are easily partitioned according to individual infants, it is natural to use Leave-One-Infant-Out Cross-Validation (LOOCV) for model selection [13]. This procedure trains 20 models, each using data from all infants, except for one left-out infant (4 sequences). We then evaluated each model by computing the average log-likelihood for the sequences from the left-out infant. The overall LOOCV score is the average of the 20 left-out log-likelihoods. This procedure directly estimates how well a model trained on all of the data generalizes to new infants. Thus, we can use LOOCV to compare HMMs built with different numbers of states. Generally, we expect that too few states will result in lower LOOCV values due to overly simplistic models and that too many states will lead to overfitting that is also detected by lower LOOCV returns.

Figure 2 shows LOOCV computed for different numbers of states (1-8). For our chosen 2-minute window size (middle curve), the 5-state model yields the best LOOCV with 6 and 7 states yielding nearly equal performance. Inspection of the observation model (see Section 5) shows that the 5-state model produced semantically distinct states, whereas the 6- and 7-state models included semantically redundant states. Thus, we selected 5 states for further analyses.

We wanted to ensure that our analysis was not overly sensitive to window size . Thus, in addition to 2-minute windows, we estimated HMM models for 1.5-minute and 2.5-minute windows and visualized the resulting models. Discretization with smaller window sizes (1-minute or shorter) is overly sensitive to instantaneous changes of the toys sets. We found that the models were qualitatively similar to the 5-state model for 2-minutes. In particular, the states between models had clear one-to-one semantic mappings.

Moreover, Figure 2 shows model-selection curves for 1.5-minute (bottom) and 2.5-minute (top) windows. Importantly, the LOOCV log-likelihoods are not directly comparable across different window sizes because window size affects both the sequence length and statistics for the observation model (e.g., count features will be biased higher/lower for longer/shorter windows). For all window sizes, 5-states achieves an LOOCV that is competitive with the best number of states. In the case of 2.5-minutes, the LOOCV curve is relatively flat after 2-states, suggesting that 2.5-minute windows may filter out significant behavior dynamics. For 1.5-minutes, the 6-state model is preferred to 5-states, but it is encouraging that the preferred number of states does not change compared to 2-minute windows. Overall, these observations suggest that our analysis is not overly sensitive to our choice of window size. The HMM implementation, which based on the Python *pomegranate* library [14], will be released in near future.



(a) (b) (c)

Figure 5. (a) Scatterplot of Δ_{bm} $vs.$ Δ_{me} for each infant, where Δ_{bm} and Δ_{me} are the change in percent of focus states from **beginning (b) to middle (m)** and from **middle (m) to end (e)**, respectively. (b) Exemplar infants from the group with decreasing then increasing focus. For each infant, the three bars correspond to the beginning, middle, and end intervals of the play session. (c) Exemplar infants from the group with increasing then decreasing focus.

V. MODEL ANALYSIS

Model Interpretation. Figure 3 depicts the 5-state HMM estimated from the data of all 20 infants. For each state, we examined the individual feature distributions (right), in order to identify potential semantic interpretations of the states. The semantic interpretations of the states are consistent with different model initialization. We identified two types of states: *focus states*, where infants focus on a relatively fixed small set of toys, and *exploration states*, where infants more rapidly select a wider set of new toys.

Focus states are characterized by feature distributions favoring lower toy-switch counts, lower toy counts, lower new-toy counts, and a higher ratio of favorite-toy playtime. We characterized 3 of the 5 states as focus states: In *Focus-1 (F1),* infants focused on a single favorite toy characterized by toy switch and toy counts always equal to one or zero; in *Focus-2 (F2*), infants focused on two toys, with one being the favorite (high favorite toy ratio). Infants mostly focused on the favorite toy, but periodically shifted to the other toy. In *Focus-set (Fset*), infants switched among a relatively fixed set of toys. This state is characterized by a high probability of having more toys and more switching than *F1* and *F2*, but still having a relatively low probability of selecting new toys.

In contrast, exploration states are characterized by higher values of toy counts, new toy counts, and toy switching. We identified two exploration states: *Explore High (E+)* is the most exploratory state with a high probability of a large toy set, frequent switching, and more new toys than the focus states. Furthermore, the fraction of time spent with a favorite toy is small. *Explore Low (E-)* typically entails less toy switching, smaller toy counts, and fewer new toys compared to *E+*. It is similar to the focus state *Fset* but introduces more new toys and has a smaller IOU, which indicates it is more novelty seeking than *Fset*.

Exploration states had the highest initial state probabilities, whereas *F1*, the most focused state, had the smallest initial state probability, indicating that infants tended to start in a more exploratory/less-focused state. Put another way, when infants encounter a new environment, they tend to investigate what is there or become familiar with the environment before engaging in focused play. Moreover, the self-transition probabilities (self-arcs for each state in Figure 3) monotonically increase with increased focus, indicating that after infants are in a focused state, they tend to remain there longer than in the exploration states.

The HMM transition diagram also shows that, at the group level, transitions from the exploration states to *F2* or *Fset* are more frequent than transitions to *F1*. Rather, the primary path to enter the most focused state *F1* is either to start there at the beginning of a session or transition there from *F2.* This finding indicates a tendency to gradually increase focus in toy selection, rather than going directly from high-exploration to high-focus.

States Over Time – Group Level. We now consider how the inferred internal behavioral states are distributed over a play session. We first computed the most-likely state for each infant during each 30-s interval by averaging the

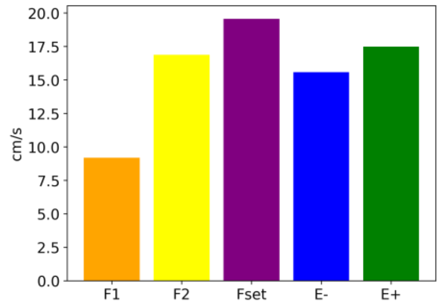


Figure 6. Average distance (cm) traveled per second over the 20 infants for each of the behavioral states.

four most likely states from four sequences (1 non-shifted and 3 shifted). We then counted the number of infants having a particular most-likely state for each 30-s interval. Figure 4 (left) shows how the number of infants in explore states (*E+, E-*) and focus states (*F1, F2, Fset*) change during the play session. In agreement with the HMM initial state distribution, initially most infants were in explore states. However, the number of infants in explore states rapidly decreased and fluctuated until only a handful were in explore states at the end. Interestingly, most infants were in focused states at the end and throughout the session.

To account for three focus states versus two explore states, we separated the contribution of *Fset* from *F1* and *F2* (Figure 4 right). We found that the number of infants in Fset is initially zero, followed by an increase and fluctuation and then decreased to zero at the end. The dominance of the focus states at the end of the play session was due exclusively to *F1* and *F2*, indicating that by the end of the session most infants entered states associated with a focus on 1 or 2 toys.

*States Over Time – Individual Differences***.** We assessed whether state transitions differed among infants. To that end, we divided the play session into equal intervals named *beginning (b), middle (m),* and *end (e)*, and measured the fraction of time each infant spent in explore versus focus states. Next, for each infant, we computed the change in percent of focus states from beginning to middle (Δ_{bm}) and from middle to end (Δ_{me}).

Figure 5a shows a scatter plot of $\Delta_{bm}\ vs.\ \Delta_{me}$ across infants. Infants tended to cluster into the upper-left or lower-right quadrants. The lower-right quadrant corresponds to increasing then decreasing focus (increasing from beginning to middle and decreasing from middle to end), whereas the upper-left corresponds to decreasing then increasing focus. Figure 5b and 5c show examples of 3 infants from each of these groups, where the percent of focus states versus exploration is shown for the 3 intervals. The observation that infants appear to form these two distinct groups invites further investigation of factors related to the grouping and whether the grouping persists in additional experiments.

Relation to Locomotion. We also examined how locomotion relates to the inferred behavior states of the HMM. Figure 6 shows the average distance (in cm) traveled per second for each state across the 20 infants. The most significant difference is that infants in the most focused *F1* state traveled substantially less (~50%) compared to other states, suggesting that infants who focus on one or no toys tend to be more stationary.

Weaker evidence suggests that infants in the *Fset* state covered more distance on average. To investigate this further, we considered the correlation across infants of state frequency and distance traveled. In particular, for each state, we computed the correlation across infants between the distance traveled and the percent of time in that state during the session (Spearman's rank). The resulting coefficients were *F1* ($r[20]=-0.01$, $p>0.2$), *F2* ($r[20]=-0.06$, $p>0.2$), *Fset* (r[20] = 0.35, $p<0.06$), *E-* ($r[20]=-0.07$, p>0.2), *E+* ($r[20] = 0.12$, $p>0.2$). Overall, there is weak statistical evidence that *Fset* has a positive correlation, while there is no evidence for the other states. This motivates further studies with larger sample sizes to assess whether infants walk more when they focus on a small toy set compared to when they select novel toys. Presumably, because the toys were designed to be used while walking, infants had more goal-oriented behavior in *Fset* compared to the exploration states. By avoiding exploratory toy selection, infants have the opportunity to travel farther distances. Yet, further studies are needed to systematically test this hypothesis.

VI. Summary and Future Work

We modeled infant behavioral states in free play with HMMs. We demonstrated an interpretable model for toy-selection that provides insights into moment-to-moment infant behavior. By analyzing state sequences across infants, we discovered trends in infants' focus on toys as sessions evolved and identified potential infant groups whose states evolved differently. In addition, we identified potential relations between behavioral states and locomotion. This study points to the promise of hidden-state modeling and analysis for other real-time behavioral data in developmental science. Moving forward, it will be important to expand the expressiveness of the hidden-state models beyond simple discrete-state HMMs. In particular, using more general Bayesian modeling paradigms to express structured priors on behavioral states may allow for more fine-tuned investigation into the structure of the states. Further, HMMs use a simplistic discrete handling of time, which required discretizing sessions into fixed-sized windows. It is important to investigate models that more powerfully handle continuous-time data. Finally, we are interested in investigating whether models of free play, such as those produced in this work, can usefully inform exploration strategies used for artificial agents in a reinforcement-learning setting.

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